ANNAMARIA IEZZI
www.aiezzi.it

INTRODUCTION TO CALCULUS

→ ETYMOLOGY

The word "calculus" comes from Latin and means "a small pebble or stone used for counting".

→ A LITTLE BIT OF HISTORY

Modern calculus was developed in the 17-th century by Newton and Leibniz independently of each other (even if there was a lot of controversy... mathematicians can be very jealous of their results!)

NEWTON: first to apply calculus to general physics.

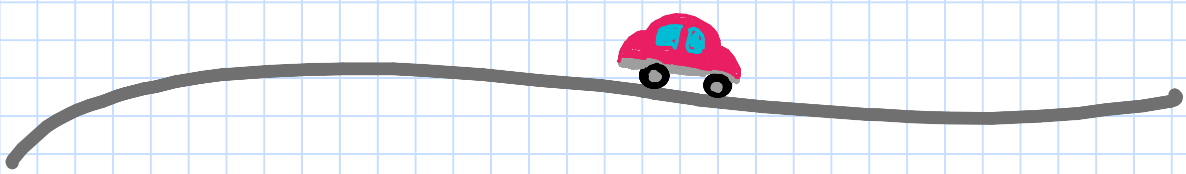
LEIBNIZ: developed much of the notation used in calculus today.

→ WHAT DOES CALCULUS STUDY?

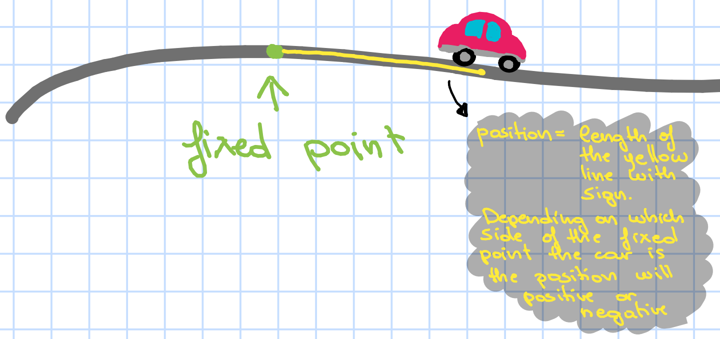
Calculus is the study of "change" and it studies change by studying "istantaneous" change (over a very small interval of time).

Let us try to understand this with an easy example: the motion of an object.

An example: the motion of an object along a fixed path



- Let us fix a point on the path. At any time we can describe the position of the object as the "distance" (with positive or negative sign) of the object from the fixed point:



We can say that the motion of an object is characterized by the sets of its numerical positions at each time

This is what we usually call a "function", which is one of the basic notions of calculus.

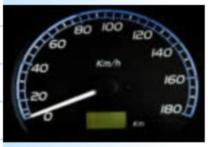
- **WHAT DOES IT CHANGE IN THIS EXAMPLE?**

  The position, i.e. the distance (with sign) of the object from a fixed point, varies with time.

- **AND HOW DOES THE POSITION CHANGE WITH TIME?**

  This depends on the velocity of the object.

AVERAGE VELOCITY VS ...



Sam and Alex are traveling in the car ... but the speedometer is broken.

Alex: "Hey Sam! How fast are we going now?"

Sam: "Wait a minute ..."

"Well in the last minute we went 1,2 km, so we are going:"

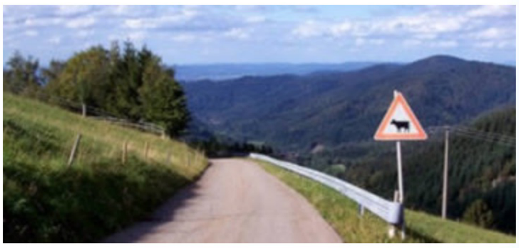
1,2 km per minute x 60 minutes in an hour = **72 km/h**

Alex: "No, Sam! Not our **average** for the last minute, or even the last second, I want to know our speed RIGHT NOW."

$$\frac{\text{displacement}}{\text{time}} = \frac{1.2\ \text{Km}}{1\ \text{min}} = \frac{1.2\ \text{Km}}{\frac{1}{60}\ \text{h}} = 1.2 \cdot 60\ \frac{\text{Km}}{\text{h}} = 72\ \text{Km/h}$$

... ISTANTANEOUS VELOCITY

Sam: "OK, let us measure it up here ... at this road sign... NOW!"



"OK, we were AT the sign for **zero seconds**, and the distance was ... **zero meters**!"

The speed is 0m / 0s = 0/0 = **I Don't Know**!

"I can't calculate it Sam! I need to know **some** distance over **some** time, and you are saying the time should be zero? Can't be done."

Here we need LIMITS!!

source: www.mathisfun.com

TWO PROBLEMS, TWO BRANCHES OF CALCULUS

1) Find the instantaneous velocity given a position function (we will see that more in general this corresponds to compute the derivative of a function).

DIFFERENTIAL CALCULUS

2) Find the position function by knowing the instantaneous velocity at all time (or, more in general, find the function by knowing it's derivative)

INTEGRAL CALCULUS

CALCULUS

DIFFERENTIAL CALCULUS

INTEGRAL CALCULUS

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FUNDAMENTAL THEOREM OF CALCULUS

This is our final GOAL!!